

TRAFFIC MODELLING FOR THE SMART CITY OF PILSEN

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Abstract

The contribution describes Traffic Modeller (www.trafficmodeller.com) - a web map application for monitoring, analysis and prediction of traffic in a metropolitan area. The technical solution of Traffic Modeller is provided. This includes the mandatory and optional input data sources (such as road network, socioeconomic data describing behavior related to transport, real traffic measurements), the algorithms processing the measurements and traffic flow modelling algorithms. As an output of Traffic Modeller, various interactive maps portraying historical, current and predicted traffic data can be obtained. The paper includes the development and deployment of Traffic Modeller application for the smart city of Pilsen.

Traffic Modeller outputs can play a crucial role in city policy-making. The resulting maps allow city planners to study the traffic behaviour while modelling various road network parameters (e.g. road closures, road capacity) and explore the effects of the changes in traffic flows in near real-time. The second part of the contribution describes typical use cases which can help policymakers in making smart, data-based and informed decisions.

Keywords: traffic modelling, Traffic Modeller, TraMod, dynamic map, web map, policy-making

1. INTRODUCTION

The objective of this paper is to present a software solution for responsive traffic modelling - Traffic Modeller. Traffic Modeller (<https://trafficmodeller.com/>) is an open-source, client-server based solution, which is being developed at the Department of Geomatics of the University of West Bohemia (<http://geomatics.zcu.cz/>) in close cooperation with SME's EDIP (<http://www.edip.cz/>) and HSRS (<http://www.bnhelp.cz/>) and the Plan4all association (<https://www.plan4all.eu/about/>). The development of Traffic Modeller is driven by a close collaboration with the City of Pilsen; namely its Department of Information Technology Administration (<https://www.sitmp.cz/>).

The key functionalities of Traffic Modeller include:

- Calculation of traffic volumes in a transport network in near real-time (depending on the size of the road network and server capacity calculation the time may vary from seconds to minutes).
- Implementation of three algorithms for the traffic assignment providing optimal calculation schema for every use case.
- Calculation a what-if analysis with user-defined scenarios which may include:
 - edit of road free-flow speed;
 - edit of road capacity, e.g. road/lane closure;
 - edit of road segment geometry, e.g. change of intersection layout;
 - addition of a new road segment, e.g. construction of a new city ring road.

The way how Traffic Modeller provides the introduced functionalities is explained further in this paper. The paper is structured as follows. Chapter State of the art describes the existing contemporary solutions and how they differ from Traffic Modeller. Chapter *Fundamentals of Traffic Modeller* includes input data, mathematical principles and algorithms used. In addition to that the way how Traffic Modeller tackles the cartographic visualization challenge is explained. The deployment of Traffic Modeller in the City of Pilsen is then described in the following chapter. The paper is concluded by the pros and cons of the designed and developed solutions, lessons learned, and the plan for future research & development.

2. STATE OF THE ART

This chapter describes the current state of knowledge, novelty and research uncertainty. First of all, the leader existing commercial solutions for traffic modelling are as follows:

- the PTV company offers a whole platform for transport and traffic modelling (<https://www.ptvgroup.com/en/solutions/products/>) including:
 - Visum for macro-modelling of traffic,
 - Vissim for microscopic and mesoscopic traffic simulation, and
 - Optima for combination of offline traffic modelling with real-time data,The PTV solutions are based on the Component Object Model.
- macro-modelling toolbox OmniTRANS (<https://www.dat.nl/en/solutions/transport-modelling-omnitrans>)
- Transport and land use modelling software Cube (<https://www.citilabs.com/software/cube/>) covering all aspects related to transport planning, engineering, and land use.
- Aimsun (<https://www.aimsun.com/>) mobility modelling software to analyse traffic from a single intersection to an entire region.
- EMME (<https://www.inrosoftware.com/en/products/emme/>) transport forecasting system for planning the movement of people.

There exist other software solutions which are less common in Europe.

Traffic Modeller provides an open-source alternative to these commercial solutions for macro-modelling of traffic. Traffic Modeller is not the only alternative. There are other research teams focusing on the scientific aspects of traffic modelling all over the world. Here follows a list of the most relevant research activities:

- DYNAMIT 2.0: a real-time model system for network management and emergency response designed by a team from MIT (<http://its.mit.edu/dynamit>) with focus on dynamic traffic models, see details in [Lu et al., 2015].
- Imperial College London design and develop Matlab toolbox for Computing of Dynamic User Equilibria (DUE) on Large-Scale Networks, see [Han et al., 2018].
- SUMO is an open source, highly portable, microscopic and continuous traffic simulation package designed to handle large road networks. It is mainly developed by the Institute of Transportation Systems at the German Aerospace Center (<http://sumo.sourceforge.net/>). The SUMO development seems to have been discontinued since 2012, see details in [Behrisch, 2011], [Krajzewicz, 2012].
- Multi-Agent Transport Simulation (MATSim, <https://www.matsim.org/>) is being developed at ETH Zurich (<https://www.ivt.ethz.ch/en/research/matsim.html>). MATSim offers a framework for demand-modelling, agent-based mobility-simulation (traffic flow simulation), re-planning, a controller to iteratively run simulations as well as methods to analyse the output generated by the modules. Details in [Horni et al., 2016].
- Transport & Mobility Leuven NV (TML, <https://www.tmleuven.be/en/>) conducts applied research to support policy decisions. Their research is focused on traffic, passenger and freight transport and the related economic impact and environmental problems. The team works on both macroscopic and microscopic traffic models.
- The Rodos project at IT4Innovations center (<http://www.it4i-rodos.cz/>) focuses primarily on online monitoring of traffic flow data and traffic management.

Although the research teams usually openly offer their results, a practical use of the results vary. The authors of Traffic Modeller (<https://trafficmodeller.com/>) believe that in comparison to the above mentioned projects, the advantage of Traffic Modeller lies in offering the following features:

- open code coming from the latest scientific research used for designing and development of the server calculations,
- advanced graphical user interface (GUI) in frontend developed according to carefully designed workflows in close cooperation with the mentioned commercial partners,
- thorough testing of the set of backend and frontend modules by the pilot user - the City of Pilsen.

The cooperation of the university research team with commercial partners and the City of Pilsen is supported by various EU and Czech research programmes. The research team started to work on this research topic in the EU CIP project OpenTransportNet in 2014 and is currently funded by two H2020 projects (PoliVisu and DUET) and a project of the Technology Agency of the Czech Republic (TRAFFO). See acknowledgment section for details.

3. FUNDAMENTALS OF TRAFFIC MODELLER

Traffic Modeller uses a traffic model to simulate traffic flows. A traffic model is understood as a mathematical model of real-world traffic which determines the volume of traffic (number of vehicles) on sections of the road network (in the model represented by a directed graph).

This chapter describes the fundamental building blocks of Traffic Modeller - blocks which are needed to build the traffic model and use it for simulations: computation of traffic flows. This includes the input data, explanation of computational principles, the traffic model itself, computation algorithms used and a description of outputs.

3.1. Input data

Similarly to any other modelling software, Traffic Modeller requires a certain type of input data. Input data can vary in detail and accuracy which later influences the reliability of the output. In this chapter, we will describe what are the mandatory input data, which data are obligatory, and how the accuracy of each data type influences the model computation. Please note that this article can not encompass all data model information. In order to examine all required attributes please see the database schema (<https://gitlab.com/kolovsky/traffic-modeler-rest/-/blob/master/database/schema.sql>).

Any traffic modelling requires two inputs:

- *Road network.* Well defined and topologically correct road network is the fundamental constraining graph structure, which describes the allowed movements within a modelled area of interest. Any topological discontinuity between road segments will result in an inaccurate model since there will be no car flow possible across that network node. Apart from the topology it is also crucial that traffic parameters of the road network reflect the real-world behaviour. Quite often the free-flow speed is confused with the speed limit which may result in unrealistic demand on such a road segment. The traffic capacities of each road segment are also crucial because it is an important part of the cost function and determines its shape. Road capacity in a city may be hard to estimate since the bottleneck of city traffic are usually crossroads and not the straight road segments themselves. However, inaccurately set segment capacity can easily result in a model where all cars are driving through a small suburban street with speed bumps or traffic lights while highways are left with no demand. With Traffic Modeller it is also possible to take special turn restrictions into account (using the so-called *turntable* described in the reference schema).
- *Traffic generators* represent the supply and demand of car trips in the previously described road network. Traffic generators are not required to have an explicit spatial component but have to be connected to one of the network nodes via a foreign key in the database. You can imagine the traffic generators as a parking lot next to an office building and the node as a driveway from the parking lot. The number of car trips (during peak hour) may come from different demographic sources and should reflect various local parameters such as a number of residents, number of schools and accessibility of public transport.. Each traffic generator may represent a city district, building block, or a single driveway – depending on the granularity of the data and the desired level of detail. With more granular traffic generators there is a trade-off with computation time.

The road network and traffic generators are the core parts of building a high quality traffic model. However, real behavior of traffic is immensely complex and it is therefore advised to enrich these minimal input requirements and use also other possibly optional data types (see [Ortuzar and Luis, 2011] for details):

- *The Origin-destination matrix* (OD matrix) consists of cells representing the number of car trips from the origin (row) to the destination (column). This shows the distribution of the aforementioned supply and demand of traffic generators across the network. If the attributes provided for traffic generators and calibration measurements are precise, the OD matrix can be estimated - however, not in real-time (usually several days are needed).
- If the OD matrix has already been calibrated then there is no need for additional *calibration measurements*. But in any other case, a calibration process should take place. The calibration process uses *calibration measurements* of traffic volumes to confront and adjust the calculated traffic model (see chapters below) to observed reality. Traffic measurements (also called traffic census) are usually acquired at neuralgic points of the traffic network. The data for such a traffic census can be gathered by various sources (ANPR cameras, road built-in sensors, manual census, ...). No matter the source, calibration measurements need further post processing steps to clean observational or systematic errors, to aggregate or harmonize observations to the same granularity and to provide statistical assessment.

3.2. Principle of computation

As it was already mentioned, the model consists of:

1. *road network* (sections of roads are represented by edges and intersections by nodes of a directed graph),
2. *traffic districts* (zones) - a defined part of the land bounded by a polygon and represented by a centroid.

The calculation of the model consists of 4 steps (see Fig. 1):

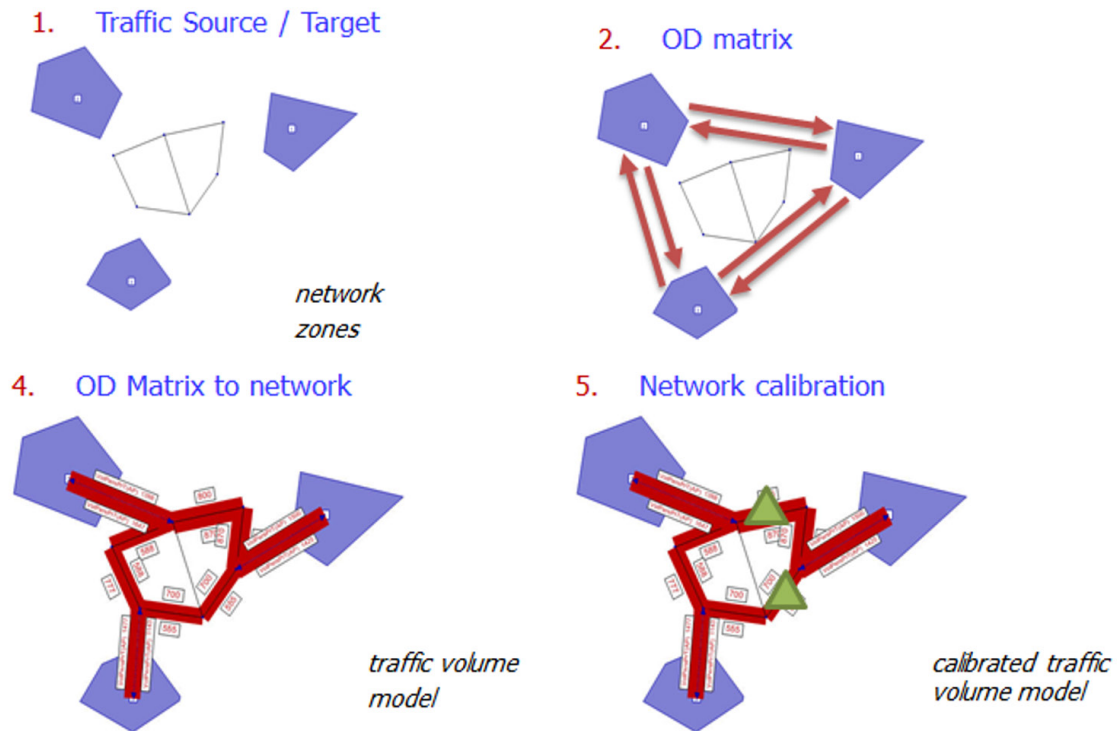


Figure 1. For steps of traffic model calculation

1. Definition of the *road network* (directed graph), traffic zones, and their centroids. Each zone has specified parameters characterizing the volume of traffic it produces.
2. Calculation *Origin Destination matrix* (OD matrix) representing the volume of traffic (number of trips) between zones. Based on the parameters of the zones and the matrix of their distances, the OD matrix is calculated (there are various calculation methods, for example the gravity model).
3. In multimodal models, the matrix is now divided according to the mode of travel (individual, public transport, bicycle, ...)
4. Assigning the *OD matrix* to the *road network* - the algorithm searches for the most convenient path over the network for each centroid relationship (more details below). The output is the traffic volume on each section of the road network.
5. The last step is to *calibrate* the model - like any mathematical model, the traffic model must be calibrated. This is a comparison of the calculated traffic volume on selected segments of the road network and the real traffic volumes (determined by the traffic survey). Both the parts of the model - road network and the OD matrix are calibrated.

3.3. Model of traffic volumes on the road network and algorithms used

A traffic model consists of two crucial sub-models: model of trip distribution and a model of traffic assignment.

Let $G = (V, E)$ is a directed graph, where V is a set of nodes and E is a set of edges. The set Z is the set of zones. The pair of zones (i, j) is called origin-destination (OD) pair and the set of OD pairs is $W = Z \times Z$. The paths between the zones create set P .

Trip distribution model

The classical gravity model is used as the trip distribution model. The distribution model estimates the number of trips between the zone i and the zone j than the number of trips is

$$Q_{ij} = O_i D_j A_i B_j f_{ij}$$

subject to

$$\sum_j Q_{ij} = O_i$$

$$\sum_i Q_{ij} = D_j$$

where O_i represents the number of trips origins in the zone $i \in Z$, D_j is the number of trips ends in the zone $j \in Z$ and the coefficients A_i and B_j are the balancing factors that are determined during the iterative proportional fitting procedure. The most important part of the equation of distribution equation is the deterrence function $f_{ij}(c_{ij})$ where c_{ij} is the cost (travel time) between the zones. The shapes of the deterrence function are described in [Ortuzar and Luis, 2011].

Traffic assignment model

The classical traffic assignment model (traffic assignment problem - TAP) is used. Let the cost c_e of the edge is dependent on traffic flow x_e

$$c_e = c_e(x_e)$$

where $c_e(x)$ is a cost function. The definition of the TAP is based on the first Wardrop's principle. This principle says: For all used paths from the source node to the destination node the travel time must be equal and this travel time must be minimal [Wardrop, 1952]. All pairs must meet with this condition.

Mathematically, the TAP can be defined as variational inequality (VI) as [Smith, 1979], [Dafermos, 1980]

$$\sum_{p \in P} c_p(x_p^*)(x_p - x_p^*) \geq 0, \forall x \in \Lambda$$

where $x = (x_p: p \in P)$ and Λ is the set of feasible flows

$$\Lambda = \{x > 0: \sum_{p \in P} x_p = Q_{ij} \forall (i, j) \in W\}$$

where Q_{ij} is one cell of ODM related to $(i, j) \in W$.

The future research will be focused on dynamic traffic assignment. The dynamic traffic assignment consists of two submodels: Dynamic User Equilibrium (DUE) and Dynamic Network Loading (DNL). Some variant of the Link Transmission model (LTM) will be used as DNL because it takes into account the queues and spillback (e.g.,

[Yperman, 2007], [Raadsen et al., 2016], [Himpe et al., 2016]). As DUE, the destination-based route-choice DUE will be used.

Algorithms for TAP solution

There are a lot of algorithms for a TAP solution. The algorithms can be split into three categories:

- Link-based algorithms. This category is represented by the Frank-Wolfe algorithm [Frank and Wolfe, 1956] and their modifications.
- Path-based algorithms. These algorithms are faster than link-based but they are memory demanding.
- Bush-based algorithms. The youngest category. These algorithms are the fastest.

Traffic Modeller implements the path-based algorithm by [Jayakrishnan et al., 1994] and bush-based B algorithm by [Dial, 2006] with some improvement by [Nie, 2010]. The implementation of the B algorithm is fully parallelized by [Potužák and Kolovský, 2020].

The B algorithm uses the decomposition of the problem to bushes. The bush is the acyclic subgraph of G related to origin zone. The bush contains only flow from the origin zone to all other destination zones. Each bush is equilibrated by shifting the flow from the minimal path to the maximal path. The minimal and maximal paths are found by topological order in the acyclic bush.

Algorithm for ODM calibration

The method by [Spiess, 1990] with some adjustments is implemented by Traffic Modeller. This method uses the steepest descent with a longstep for the object function minimization. Some adjustments are described in [Kolovský et al., 2018]

Implementation

All the algorithms are implemented in Java or Scala programming language. Whole application is based on Spring Boot and PostgreSQL with extension PostGIS.

3.4. Outputs and visualizations

In common (desktop) traffic modelling software (Visum, CUBE, OmniTrans) the emphasis is on outputs for traffic engineers. The most used graphic expression is the traffic flow map. The thickness of the line corresponds to the volume of traffic (Fig. 2)

However, such outputs are difficult to understand (and not attractive) to the general public and political representation. They prefer illustrative - colorful and moving expressions. Therefore, an approach was chosen that will allow presentation using a web application, and thus provide:

1. easy operation, accessibility,
2. dynamics - moving on the timeline,
3. better understanding - in addition to the volume of traffic (number), the level of service (exhaustion of capacity) is also displayed in a color scale.

The transfer of the outputs of the traffic model to the Internet brought not only positive responses but also some problems.

- the general public's approach has resulted in higher demands for the model to agree with reality,
- the capacity of the road (and thus its color at a certain traffic intensity) is inaccurate - the capacity of the road network in the city is given more capacity of intersections, which, however, the model can not accurately take into account.

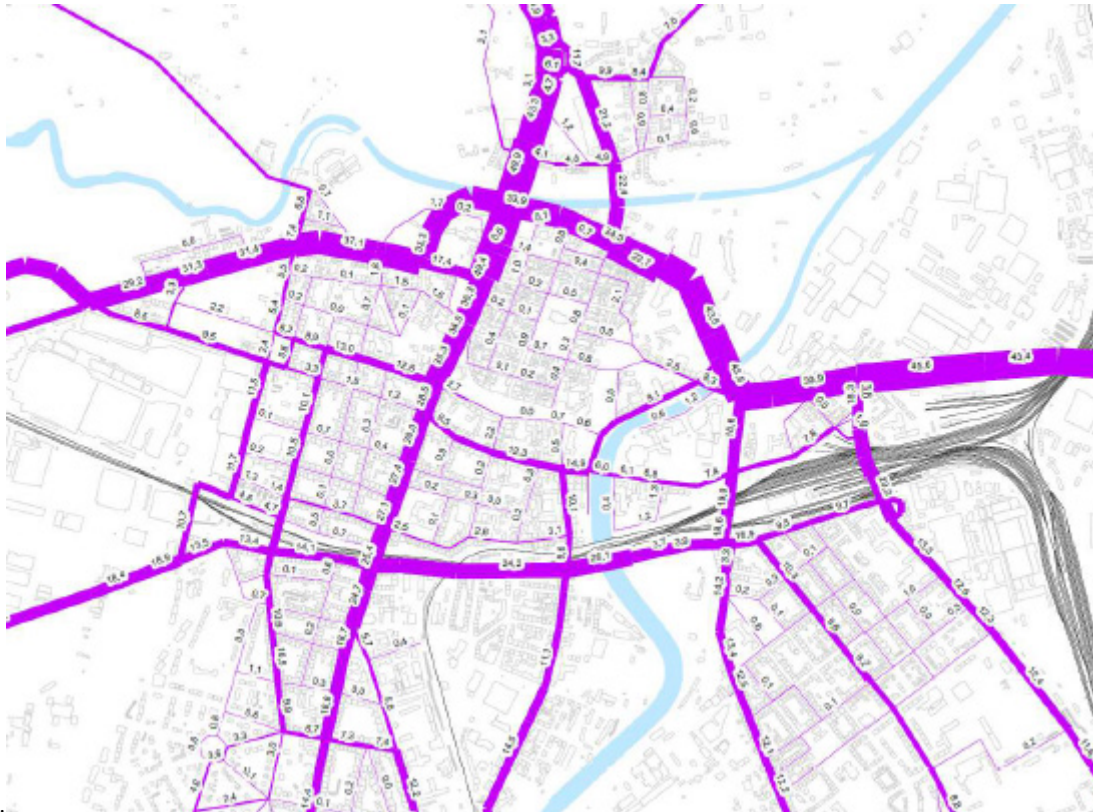


Figure 2. Traditional traffic flow map (center of the Pilsen city)

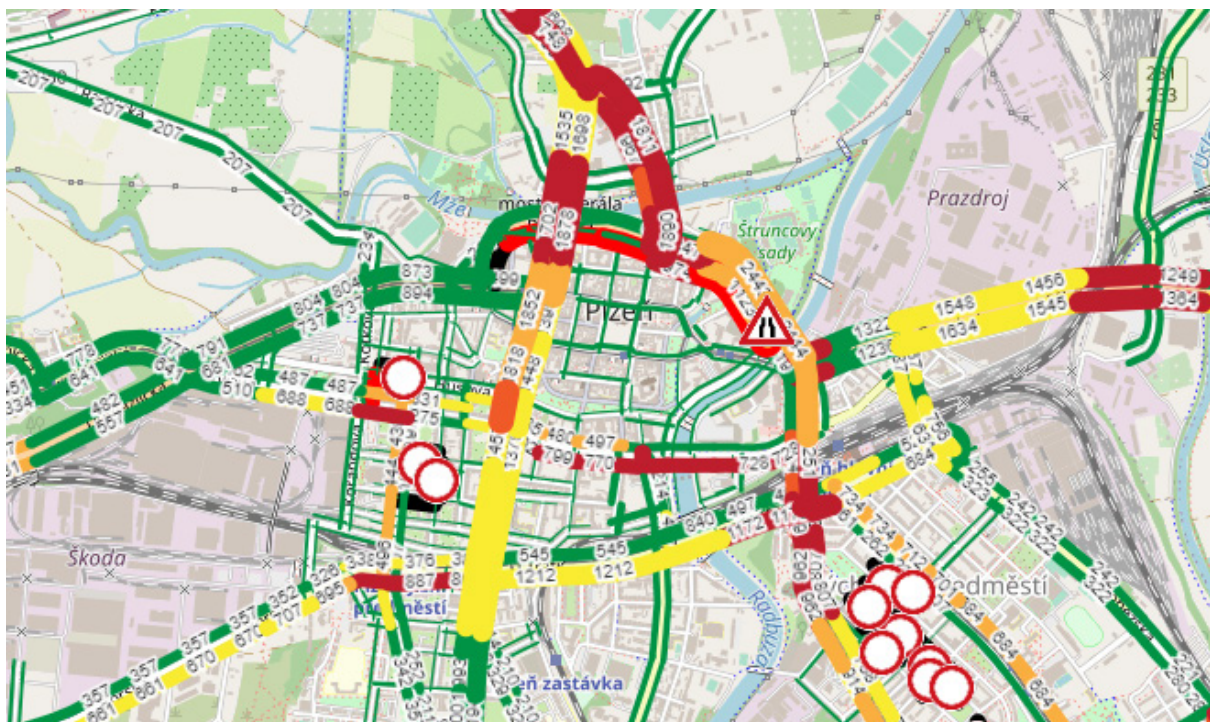


Figure 3. Traffic flow map in Traffic Modeller (center of Pilsen city)

Daily (as well as weekly and annual) variations in traffic volume were used to move the model over time. These were processed from sensor data located in the roadway at each intersection controlled by traffic lights. The data allows you to calculate how road traffic changes during the day, week and year. A time slice of traffic flow map in Traffic Modeller is shown in figure 3. Next, figure 4 shows the daily variations of traffic volume.

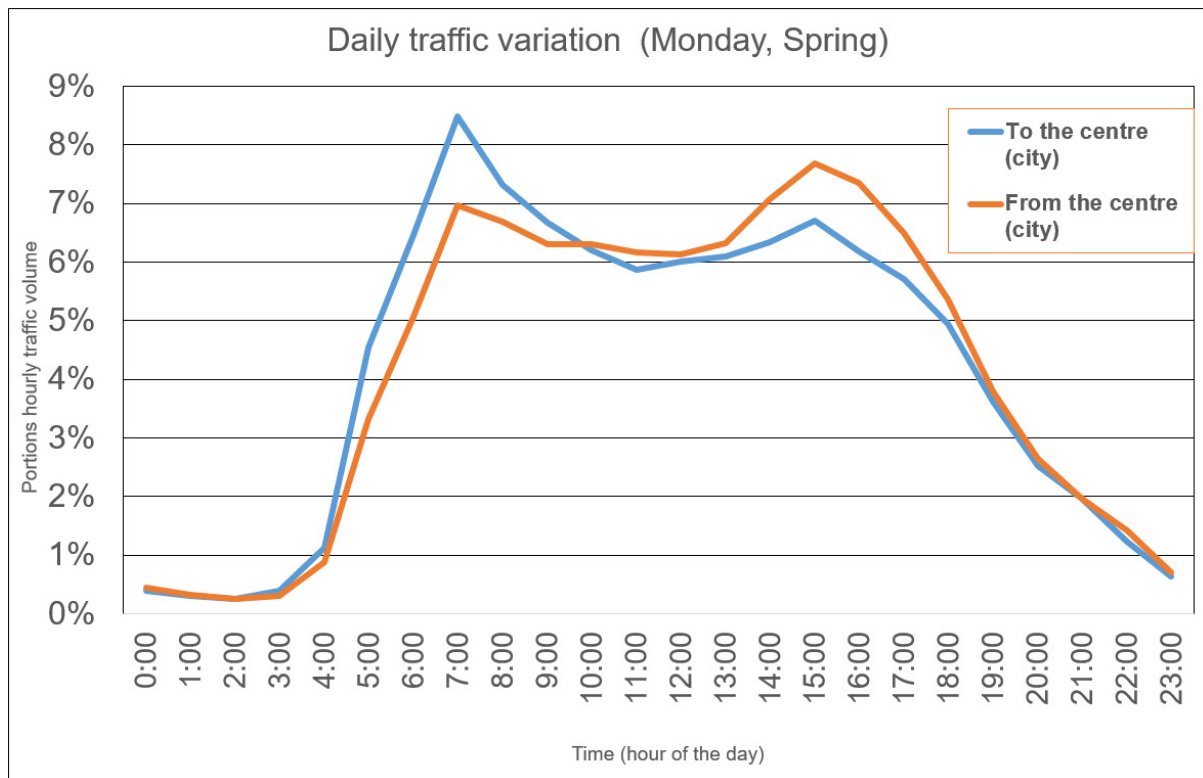


Figure 4: Daily variations in traffic volume

4. THE GOOD PRACTICE EXAMPLE IN THE SMART CITY OF PILSEN

As stated in the previous chapter, Traffic Modeller development and testing would not be possible without reliable traffic data. The first large pilot region became the city of Pilsen. Pilsen is a mid-sized city in West of Bohemia with a population around 170 thousand people and approximately 530 cars per 1000 person of population. Without full outside traffic ring the city center tracks a lot of transit traffic, especially between north and south of the city. All of which contributes to annual average daily traffic to be around 20 thousand in one direction of travel for the most frequented roads in the inner city. (<http://www.svsmp.cz/dopravni-pruzkumy/informace-o-doprave-v-plzni-za-rok-2018.aspx>)

The default traffic model for the city of Pilsen consists of 339 traffic generators shaping 339 x 339 OD matrix, 10 533 road segments and 4 723 nodes. The model was designed and then calibrated using a dataset of 250 million observations from 627 road built-in sensors, with a 90 seconds granularity in time. A whole year period was processed in order to provide capture traffic variations during different hours, days in a week and even seasons.

Firstly, Traffic Modeller Pilsen Web App (see the public version - without the experiment modelling functionality - here: <https://intenzitadopravy.plzen.eu/>) offers Pilsen policymakers web visualization of the Pilsen default traffic model (Fig. 5 a - Default traffic model). This model is the result of an in-depth calibration and correction so that it as closely as possible reflects the average weekday peak hour. This model can be additionally visualized in the form of any specific hourly traffic volume (e.g. June, Sunday from 5 pm to 6 pm).

The core of Traffic Modeller Web App is however the computation and visualization of user-defined traffic experiments. The user story for experimenting with the traffic model is as follows: the user creates an experiment that acts as a container for at least one, but usually more traffic events. Once an experiment is created the user can further create a new event. When creating a new event user can first fill some optional attributes (name, description, and from/to date) and then by clicking in the map selecting those road segments which attributes are to be changed for the experiment. Once segments are selected, users can then choose if free-flow speed and/or capacity of those segments should be altered. When these edits are saved, the experiment can be modelled and within a couple of seconds users can see the impact of tested changes in the network. (Fig. 5 b - Experiment traffic model) Afterward, this process can be easily repeated with slightly changed parameters To allow better comparison between various experiments, Traffic Modeller Web App also offers special visualization, *Differential traffic map*, which shows the difference in traffic

volumes (Fig. 5 c - Difference in traffic volumes). City policymakers might for example decide that closing the whole street entirely is not ideal and that 50% capacity for a longer time period is a better solution for that particular road reconstruction.

The above-described user story represents the core of Traffic Modeller Web App, however, as city traffic is not operated by a single user we had to extend this user story with additional functionalities. These functionalities were not part of our original design but were added during development thanks to user testing done by city employees at the information technology department at the city of Pilsen. Those user demanded functionalities are:

User management with user roles for adding new events only without the option to experiment with the model needed for other public organizations (e.g. waterworks department) that may request but have no need for traffic modelling it self; visualization of live traffic from sensors, storing, and visualization of historical hourly traffic volumes from sensors (these historical volumes can also be compared using the Differential traffic map).

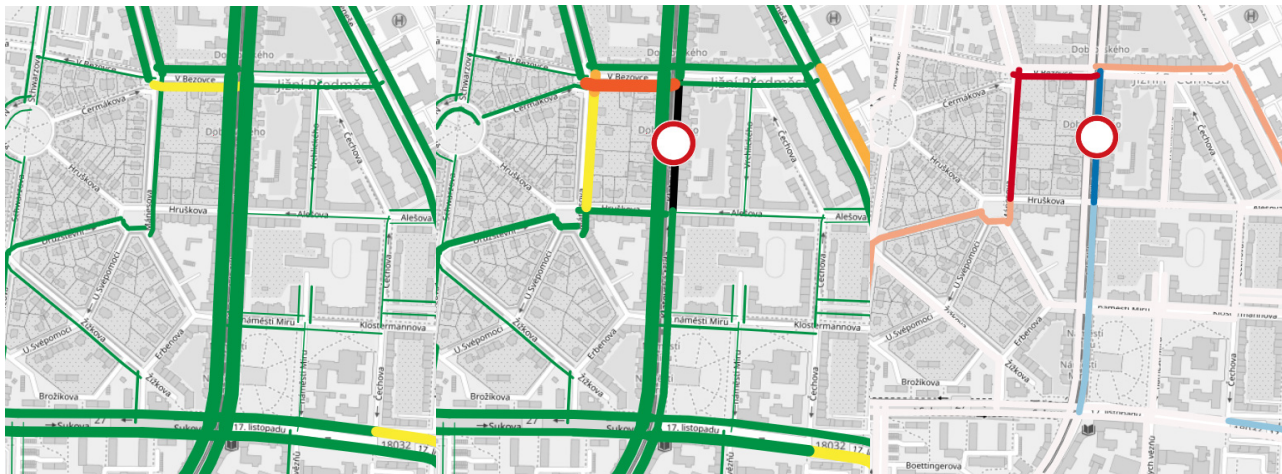


Figure 5. a) Default traffic model on the left, b) Experiment traffic model in the middle, c) Differential traffic map on the right.: red - more traffic after closure, blue - less traffic after closure
(Please note that despite only streets with significant traffic being visible, all streets are used for the traffic modelling)

CONCLUSIONS AND FUTURE RESEARCH

The contribution introduces the current state of geotechnologies used for traffic modelling. Traffic modelling is a crucial part of the smart city and digital twin concepts. The fundamentals of the Traffic Modeller are described and a good practice example is demonstrated in the smart city of Pilsen. The authors have learned the following lessons which will be reflected in the future research and developments.

Traffic modelling is extremely sensitive to relevant input data. In order to calculate a good quality traffic model, it's important to have:

- a proper information from traffic generators from which the origin-destination matrix is calculated,
- an accurate shape of the modelled network,
- allowed (respectively prohibited) turns in junctions properly set,
- each network segment (and ideally also each junction) set with adequate capacity and free flow speed,
- a robust traffic census dataset for calibration.

The results as presented in the Pilsen use case are close to reality and are used for data driven decisions in policy making. Please note that an updated information about the status of the Traffic Modeller for the city of Pilsen is maintained at <https://plzen.trafficmodeller.com>.

The future research and development of the Traffic Modeller will focus firstly on the implementation of algorithms for dynamic traffic assignment, as mentioned at the end of Section Traffic assignment model. The next step will be to improve the user experience by redesigning the frontend application to streamline the workflows and graphical user interface. This turns out to be a cutting edge for the success of the Traffic Modeller. User experience, especially of the policy makers, can help in spreading the software in the future. The future research and development include extensions of the Traffic Modeller calculating noise and pollutants produced by road transport.

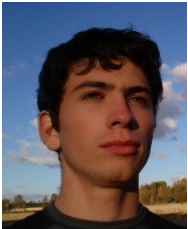
BIOGRAPHY



Karel Jedlička: The theoretical background of Karel's research lies in modeling, analysis, and even simulation using multidimensional (geographic) data structures. In particular, Karel actively researches on 3D and 4D aspects of Geographical Information Systems. Karel focuses on influence on transport to the city life, by designing and developing interactive traffic models for Digital twins of Smart cities. Karel has been leveraging his skills in various EU projects since 2007. Karel usually acts as a leader of small research or technical team in the project.



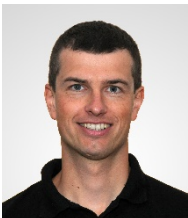
Daniel Beran graduated from the University of West Bohemia (UWB) at Faculty of Geomatics in 2018 with an Ing. degree (eq. to MSc). Since 2014 he has been active in various H2020 projects (OpenTransportNet, PoliVisu, DUET). In 2015 Daniel joined the Traffic Modeller team working as data manager and GIS analyst. Since 2018 he is undergoing a PhD study program at the UWB exploring classification of LiDAR point clouds. In 2018 and 2020 Daniel was a guest researcher at TU Delft and TU Wien respectively. He is a part time GIS teacher at Pilsen highschool of civil engineering. He has experience in GIS, traffic modelling, 3D visualization and point cloud processing.



František Kolovský focuses on transportation research especially on Dynamic Traffic Assignment. He has also strong background in time-dependent shortest path search, graph algorithms and in open GIS. He likes to use the following technologies as a research tool: Scala, Python, Java, PostGIS and Apache Spark.



Jan Martolos is a traffic engineer specializing in mathematical modeling in transportation. He studied mathematics at the University of West Bohemia in Pilsen and traffic engineering at the Czech Technical University in Prague. He deals mainly with traffic surveys, traffic models, the capacity of intersections and research in transport.



Tomáš Mildorf is a project manager at the Department of Geomatics at the University of West Bohemia. He is an experienced manager of large EU projects ensuring the sustainability of the research and development activities at the department, including the exploitation of Traffic Modeller. He was involved in numerous EU projects including Plan4all, Plan4business, OpenTransportNet, SDI4Apps, DataBio and AFarCloud.



Michal Kepka is an academic staff member at the Department of Geomatics at the University of West Bohemia. Michal focuses on GIS, sensor data, spatial data modelling and visualization. He is an experienced researcher and has been involved in several EU projects in the last 10 years - plan4business, SDI4Apps, DataBio, AFarCloud, Peregrinus Silva Bohemica.



Jan Šáněl: Jan's professional background bridges geography and information technologies. Jan graduated from Geographical cartography and geoinformatics at Masaryk University in 2019. Jan works as a software developer focusing on web mapping technologies (OpenLayers, Leaflet, Turf.js). He worked on optimization of HSLayers - a web mapping platform and now is the main front end developer of the Traffic Modeller.

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- TRAFFO: Innovative Approaches to Mathematical Traffic Modelling for Sustainable Development of Cities and Regions. The Technology Agency of the Czech Republic. Programme: DOPRAVA 2020+. Grant agreement no: CK01000096

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